

Module-5: Communication SYSTEMS

①

Syllabus: Introduction, Elements of Communication Systems, Modulation: Amplitude Modulation, Spectrum Power, AM Detection (Demodulation), Frequency and Phase Modulation. Amplitude and Frequency Modulation: A Comparison.

* Communication: Communication is the process of exchanging information between two points (through connection (Wired) @ Connectionless (Wireless) medium)

* Communication System: Communication System is a set of electronic equipments used for communication purpose.

* Elements of Communication Systems @ (Block diagram of Communication System):

Fig ① Shows the basic elements of a Communication System

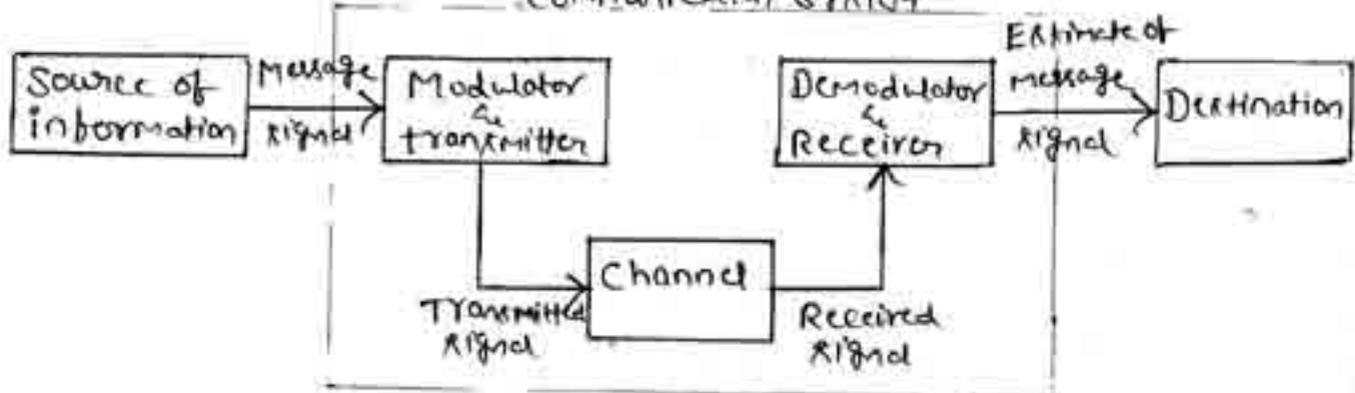


Fig ①: Block diagram of Communication System

Source of information:

→ The four important sources of information are:
① Speech ② Television ③ Facsimile (Fax) & ④ Personal computer

→ The signal that carries information cannot travel long distance.

Modulator & transmitter:

→ It converts the information signal to a form suitable

for transmission over the channel (Modulation)

→ It amplify the information signal.

Channel:

→ It is the physical medium through which the information is sent. (It may introduce noise & distortion)

→ It can be a pair of conducting wires, optical fiber, co-axial cable, waveguide @ free space.

Demodulator & Receiver:

→ It extracts the message signal from the received signal (Demodulation @ Detection)

→ It amplify the message signal. (Remove noise & filtering)

Destination:

→ The message signal is fed to Loud speaker, TV picture tube, Computer display screen etc

Note:

① Message signal is called as Information signal @ Baseband signal @ Unmodulated signal @ Low frequency signal @ Modulating signal

② Carrier signal is called as High frequency signal

③ Frequency range with application

Sl. no.	Frequency range (Band)	Applications
1	30Hz - 300Hz, Extremely Low frequencies (ELF)	Power transmission
2	300Hz - 3KHz, Voice frequency (VF)	Audio, submarine communication, navigation

3	3KHz - 30KHz, Very Low frequencies (VLF)	Submarine Communication, Navigation
4	30KHz - 300KHz, Low frequencies (LF)	Submarine Communication, Navigation.
5	300KHz - 3MHz, Medium frequencies (MF)	AM broadcast, Aeronautical Comm.
6	3MHz - 30MHz, High frequencies (HF)	Shortwave transmission.
7	30MHz - 300MHz, Very high frequencies (VHF)	TV & FM broadcast.
8	300MHz - 3GHz, ultra high frequencies (UHF)	Communication Satellites, Cellular Phones, Personal Comm Systems
9	3GHz - 30GHz, Super high frequencies (SHF)	Satellite communication, Radar
10	30GHz - 300GHz, Extremely high frequencies (EHF)	Satellite communication, Radar

* Need for modulation:

- ① Increases the operating range @ Increases the range of communication @ Long distance communication:

Modulation increases the frequency of the signal & thus they can be transmitted over long distances.

- ② Reduce the height of antenna:

Minimum height of the antenna is,

$$h = \lambda/4 = \frac{c}{4f} \quad \left(\because \lambda = \frac{c}{f} \right)$$

If $f = 1\text{KHz}$, $h = \frac{3 \times 10^8}{4 \times 1 \times 10^3} = 75,000\text{m}$

(Impractical)

$\lambda \rightarrow$ Wavelength (m)
 $c \rightarrow$ Velocity of light
 ($3 \times 10^8 \text{ m/s}$)
 $f \rightarrow$ Frequency (Hz)

(4)

If $f = 1\text{MHz}$, $h = \frac{3 \times 10^8}{4 \times 10^6} = 75\text{M}$ (Practical)

③ Avoid mixing of signals:

Modulation avoids mixing of signals (by using different carrier frequencies for different signals)

④ Allows multiplexing of signals:

Modulation allows the transmission of two or more signals simultaneously over the same channel (multiplexing)

⑤ Improve the Quality of reception & Reduces noise:

Modulation reduces the effect of noise & thus improves the quality of reception

⑥ Allows adjustments in bandwidth

Modulation allows to vary the bandwidth of the signal (modulated signal)

⑦ Wireless Communication:

Modulation avoids the use of wires (sometimes), i.e. signal can be radiated into free space

* Modulation:

The process of changing one of the characteristics (E.g. Amplitude, frequency & Phase) of a carrier signal (High frequency signal) with respect to the instantaneous values of the message signal (Low frequency signal) is called modulation.

Note: ① The message signal is given by,

$$m(t) = V_m \sin \omega_m t$$

Where $V_m \rightarrow$ Peak @ maximum Amplitude of message signal
 $\omega_m \rightarrow$ Angular frequency of the message signal.

② The carrier signal is given by,

$$C(t) = V_c \sin(\omega_c t + \phi)$$

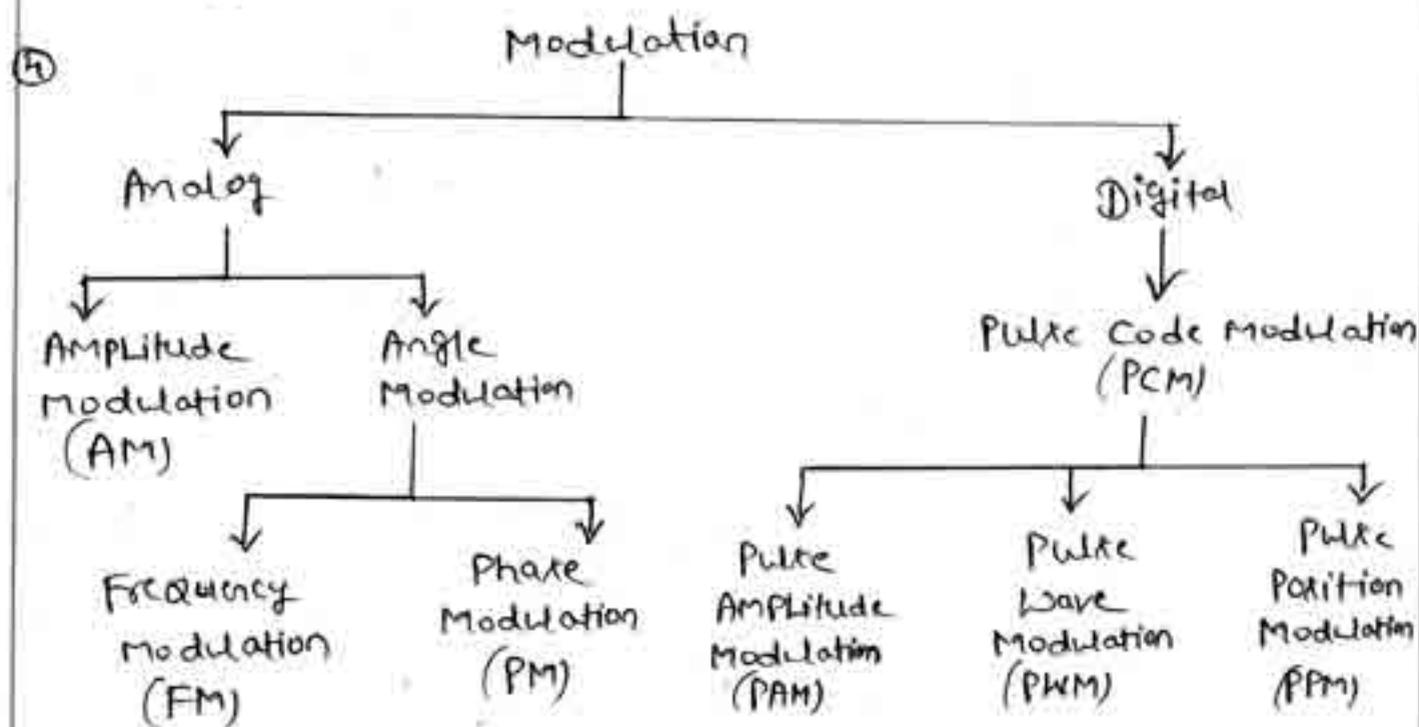
Where, $V_c \rightarrow$ Peak @ maximum Amplitude of carrier signal.

$\omega_c \rightarrow$ Angular frequency of the carrier signal

$\phi \rightarrow$ Phase angle of the carrier signal with respect to some reference.

③ Demodulation @ detection

The process of recovering (extracting) the information signal (message signal) from the modulated signal is called demodulation @ detection



Types of Modulation:

There are three basic types of modulation.

① Amplitude Modulation

② Frequency Modulation } Angle Modulation
③ Phase Modulation }

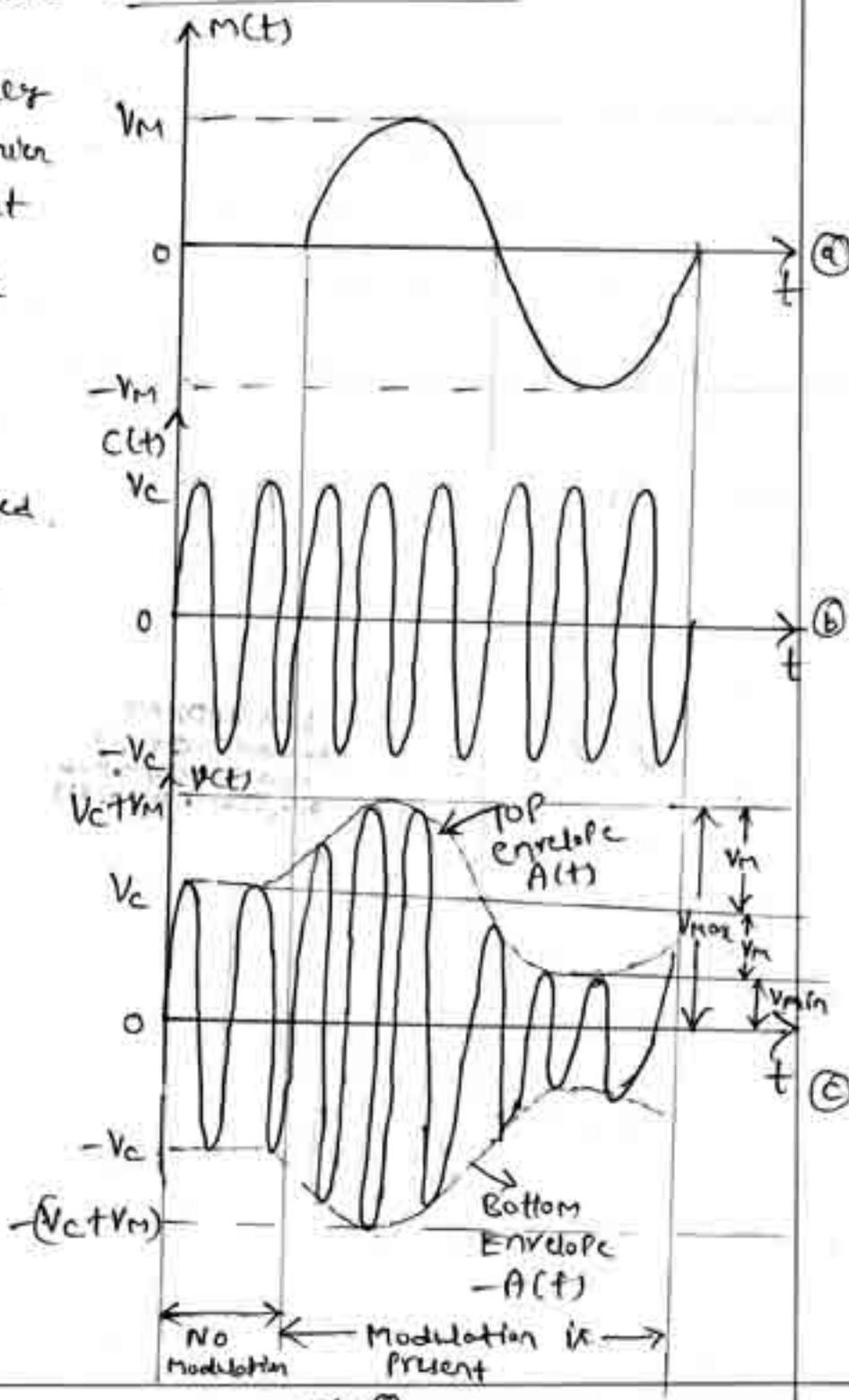
① Amplitude Modulation (AM):

The process of changing the amplitude of carrier wave with respect to the instantaneous values of the message signal is called Amplitude Modulation (AM)

→ Here the frequency & phase of the carrier wave is kept constant

→ During the positive half-cycle of the message signal, the amplitude of the carrier wave is increased.

→ During the negative half-cycle of the message signal, the amplitude of the carrier wave is decreased.



$f_{USB} - f_{LSB}$

$\omega_{USB} - \omega_{LSB}$

ie $BW = (f_c + f_m) - (f_c - f_m) \text{ or } (\omega_c + \omega_m) - (\omega_c - \omega_m)$

$BW = 2f_m \text{ (Hz)}$ - 8

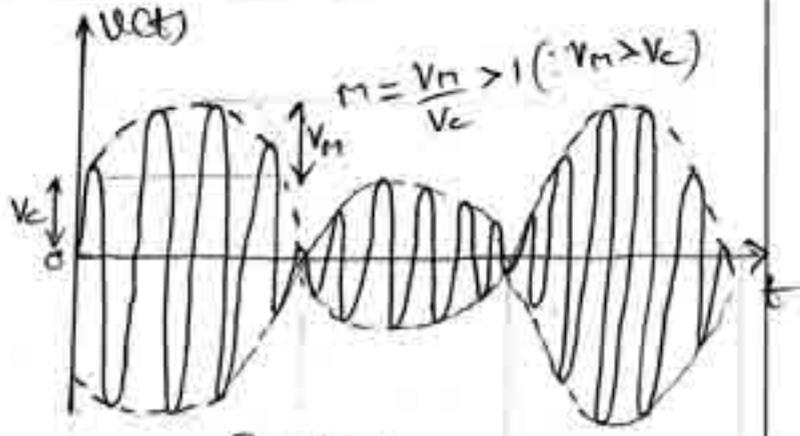
$BW = 2\omega_m \text{ (rad/s)}$ - 9

5) Effect of Modulation index on AM

Case (i): $(m > 1)$

→ The carrier is said to be over-damped

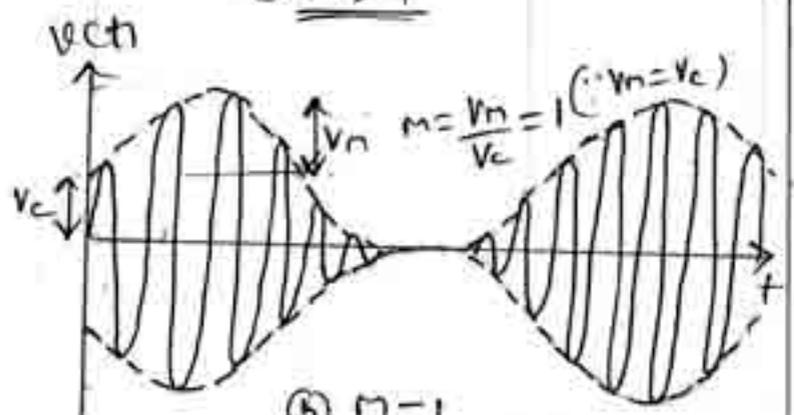
→ AM wave is distorted (Clipped off)



(a) $m > 1$

Case (ii): $(m = 1)$

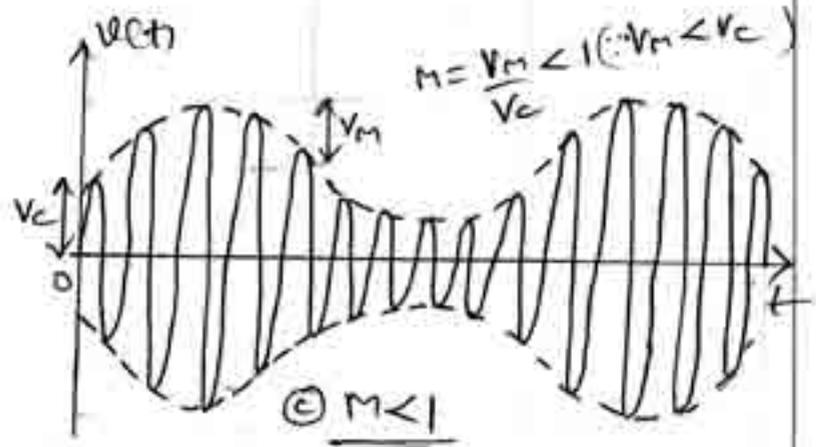
→ The carrier is said to be critically damped



(b) $m = 1$

Case (iii): $(m < 1)$

→ The carrier is said to be under-damped



(c) $m < 1$

∴ Modulation index should not exceed 1

$\therefore m \leq 1$

Fig (4): Effect of Modulation Index (m) on AM

6) Expression for Modulation Index of AM (in terms of V_{max} & V_{min})

From fig (1)(c), we can write

$$V_{max} = 2V_m + V_{min}$$

$$\Rightarrow 2V_m = V_{max} - V_{min}$$

$$\Rightarrow V_m = \frac{V_{max} - V_{min}}{2} \quad \text{--- (10)}$$

$$\& V_c = V_m + V_{min} \quad \text{--- (11)}$$

Using (10) in (11), we get

$$V_c = \frac{V_{max} - V_{min}}{2} + V_{min}$$

$$V_c = \frac{V_{max} + V_{min}}{2} \quad \text{--- (12)}$$

$$\text{LRT } m = \frac{V_m}{V_c} \quad \text{--- (13)}$$

Using (10) & (12) in (13), we get

$$m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

$$m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

(14)

$$M = \frac{V_{max} - V_{min}}{V_{max} + V_{min}} \times 100\%$$

(15)

* Expression for total average power of a AM wave

1) Spectrum power in AM wave

AM wave is given by,

$$V(t) = \underbrace{V_c \sin \omega_c t}_{\text{Unmodulated Carrier}} + \underbrace{\frac{mV_c}{2} \cos(\omega_c - \omega_m)t}_{\text{Lower Sideband}} - \underbrace{\frac{mV_c}{2} \cos(\omega_c + \omega_m)t}_{\text{Upper Sideband}}$$

The total power P_t is given by,

$$P_t(\text{total}) = P_{ca}(\text{carrier}) + P_{LSB}(\text{LSB}) + P_{USB}(\text{USB}) \quad \text{--- (16)}$$

$$\Rightarrow P_t = \frac{V_{car}^2}{R} + \frac{V_{LSB}^2}{R} + \frac{V_{USB}^2}{R} \quad \text{--- (17)} \quad \left[\because \text{Power } P = \frac{V^2}{R} \right]$$

Where, V_{car} → RMS Value of carrier @ Unmodulated carrier
 V_{LSB} → RMS value of Lower sideband
 V_{USB} → RMS value of upper sideband,
 R → Resistance

$$\therefore V_{car} = \frac{V_c}{\sqrt{2}} \quad V_{LSB} = \frac{mV_c}{2\sqrt{2}} \quad V_{USB} = -\frac{mV_c}{2\sqrt{2}} \quad \left(\because V_{rms} = \frac{V_{max}}{\sqrt{2}} \right)$$

- (18)
- (19)
- (20)

Using (18), (19) & (20) in (17) we get.

$$P_t = \frac{(V_c/\sqrt{2})^2}{R} + \frac{(mV_c/2\sqrt{2})^2}{R} + \frac{(mV_c/2\sqrt{2})^2}{R}$$

$$\Rightarrow P_t = \frac{V_c^2}{2R} + \frac{m^2}{4} \frac{V_c^2}{2R} + \frac{m^2}{4} \frac{V_c^2}{2R}$$

$$P_t = \frac{V_c^2}{2R} \left(1 + \frac{m^2}{4} + \frac{m^2}{4} \right)$$

$$P_t = \frac{V_c^2}{2R} \left(1 + \frac{m^2}{2} \right) \quad (w) \text{ (21)}$$

- (21)

$$P_t = P_{car} \left(1 + \frac{m^2}{2} \right) \quad (w) \text{ (22)}$$

Where $P_{car} = \frac{V_c^2}{2R}$

Eqn (21) & (22) is the expression for total power in AM wave

Note:

- ① From (21) & (22) we can say, (a) Total Power in AM wave is more than carrier power
- (b) Total Power depends on the modulation index m .

② Maximum Power in an AM wave

From eqn (22), $P_t = P_{car} \left(1 + \frac{m^2}{2} \right)$
 Maximum value of m is 1

$\therefore P_t = P_{con} (1 + \frac{1}{2})$

$P_t = 150\% \text{ of } P_{con} \Rightarrow P_t = 1.5 P_{con} // \text{--- (23) } \odot P_{con} = 66.66\% P_t$

Thus the maximum power in AM wave is $1.5 P_c$ @ 150% of the carrier power P_{con}

③ Modulation index in terms of carrier power (P_{con})

↳ total power of AM wave (P_t)

From eqn (22), $P_t = P_{con} (1 + \frac{m^2}{2})$

$\Rightarrow \frac{P_t}{P_{con}} = 1 + \frac{m^2}{2}$

$\Rightarrow \frac{m^2}{2} = \frac{P_t}{P_{con}} - 1$

$\Rightarrow m^2 = 2 \left(\frac{P_t}{P_{con}} - 1 \right)$

$\Rightarrow m = \sqrt{2 \left(\frac{P_t}{P_{con}} - 1 \right)} // \text{--- (24)}$

④ Current relation in AM wave (Voltage relation)

From eqn (22), $P_t = P_{con} (1 + \frac{m^2}{2})$

$\left[\begin{aligned} &= VI \text{ } \odot \\ \therefore \text{ Power } P &= I^2 R \text{ } \odot \\ &= V^2 / R \end{aligned} \right]$

$\Rightarrow I_t^2 R = I_{con}^2 R (1 + \frac{m^2}{2})$

$\Rightarrow I_t = I_{con} \sqrt{1 + \frac{m^2}{2}} \text{--- (25)} \quad I_{con} = \frac{I_t}{\sqrt{1 + \frac{m^2}{2}}} \text{--- (26)}$

$m = \sqrt{2 \left(\frac{I_t^2}{I_{con}^2} - 1 \right)} // \text{--- (27)}$

$M = \sqrt{2 \left(\frac{V_t^2}{V_{con}^2} - 1 \right)}$

⑤ Modulation index & total power when a carrier is amplitude modulated by several sine waves :

Let $V_1, V_2, V_3 \dots$ etc be amplitudes of sine waves, Then the total modulating voltage V_t is,

$$V_t = \sqrt{V_1^2 + V_2^2 + V_3^2 + \dots}$$

$$\Rightarrow \frac{V_t}{V_c} = \frac{\sqrt{V_1^2 + V_2^2 + V_3^2 + \dots}}{V_c}$$

$$\Rightarrow \frac{V_t}{V_c} = \sqrt{\left(\frac{V_1}{V_c}\right)^2 + \left(\frac{V_2}{V_c}\right)^2 + \left(\frac{V_3}{V_c}\right)^2 + \dots}$$

$$\boxed{m_t = \sqrt{m_1^2 + m_2^2 + m_3^2 + \dots}} \quad \text{--- (28)} \quad (m_t \leq 1)$$

Where, $m_t = \frac{V_t}{V_c} \rightarrow$ total modulation index

$m_1 = \frac{V_1}{V_c}, m_2 = \frac{V_2}{V_c} \dots \rightarrow$ Individual modulation indices

From eqn (28), the total modulation index is the square root of the sum of the squares of the individual modulation indices.

The total power in the AM wave is,

$$\boxed{P_t = P_{cos} \left(1 + \frac{m_t^2}{2}\right)} \quad \text{--- (29)}$$

⑥ Transmission efficiency of AM wave :

The ratio of the power carried by the sidebands (P_{sb})

(or power that contains information) to the total power (P_t) is called transmission efficiency or efficiency (η)

Transmission efficiency is,

$$\eta = \frac{P_{sB}}{P_t} \quad \text{--- (30)}$$

$$= \frac{P_{LSB} + P_{USB}}{P_t}$$

$$= \frac{\frac{M^2}{4} \frac{V_c^2}{2R} + \frac{M^2}{4} \frac{V_c^2}{2R}}{P_t}$$

$$\frac{V_c^2}{2R} \left(1 + \frac{M^2}{2}\right)$$

$$\frac{V_c^2}{2R} \left(\frac{M^2}{4} + \frac{M^2}{4}\right)$$

$$\frac{V_c^2}{2R} \left(\frac{M^2}{2}\right)$$

$$\eta = \frac{\frac{M^2}{2}}{1 + \frac{M^2}{2}}$$

$$\begin{aligned} \therefore P_{LSB} &= \frac{V_{LSB}^2}{R} = \frac{\left(\frac{MV_c}{2/\sqrt{2}}\right)^2}{R} \\ P_{USB} &= \frac{V_{USB}^2}{R} = \frac{\left(\frac{MV_c}{2/\sqrt{2}}\right)^2}{R} \\ P_t &= \frac{V_c^2}{2R} \left(1 + \frac{M^2}{2}\right) \end{aligned}$$

$$\eta = \frac{M^2}{2 + M^2} \quad \text{--- (31)}$$

$$\% \eta = \frac{M^2}{2 + M^2} \times 100\% \quad \text{--- (32)}$$

From (31) & (32), it is clear that the transmission efficiency increases with modulation index.

(i) When $M=0$, $\eta = \frac{0}{2+0} = 0$

(ii) When $M=0.5$, $\eta = \frac{0.5}{2+0.5} = 11.1\%$

(iii) When $M=1$, $\eta = \frac{1}{1+2} = 33.33\%$ (Max Value of M is 1)

\therefore Higher the modulation index, higher is the transmission efficiency. \therefore Maximum efficiency in AM is 33.33%.

* AM detection (demodulation) (Envelope detector):

→ The process of recovering the message signal (audio signal) from the modulated signal is known as demodulation or detection (ie demodulation is the reverse of the modulation)

→ Fig 5 shows the envelope detector. It consists of a diode followed by a RC circuit.

→ Demodulation of AM involves two steps

- ① Diode eliminates the bottom envelope
- ② RC circuit (LPF) removes the high frequency carrier

→ The signal $V(t)$ is passed through a diode to cut off the bottom half (Lower half)

→ During positive half-cycle of the input, the diode is forward biased & the capacitor 'C' charges to peak value of the input signal. When the input signal falls below the peak value, the diode becomes reverse-biased & the capacitor 'C' discharges through the load resistor 'R'. The discharging process continues until the

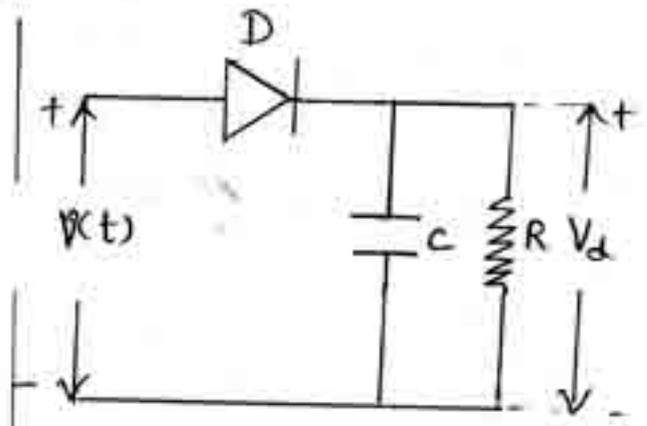


Fig 5: Envelope detector

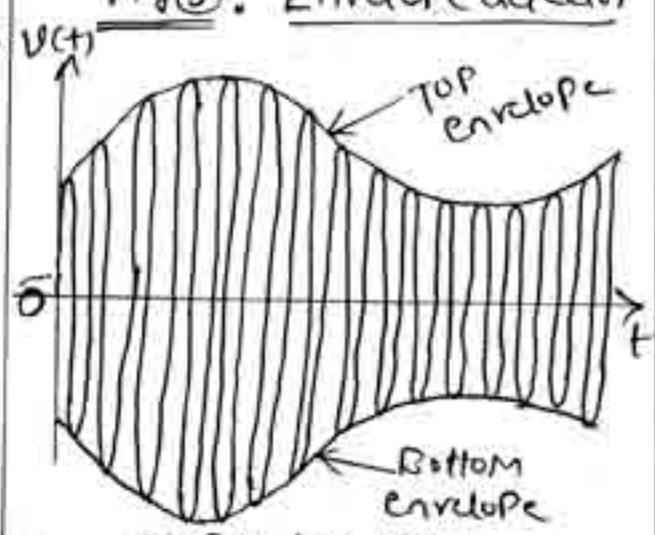


Fig 6: AM signal

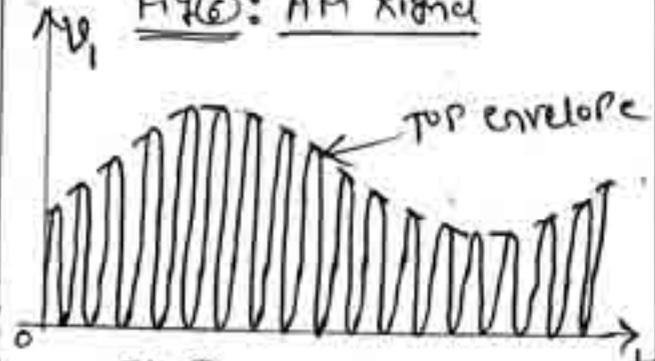


Fig 7: Rectified output

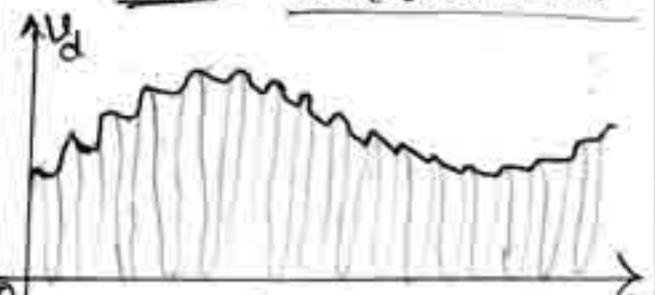


Fig 8: AM detector output

next ~~is~~ Positive half cycle.

→ When the input signal becomes greater than the voltage across the capacitor, the diode conducts again as the process is repeated. (Old V_C follows the modulated wave envelope)

→ The time constant (RC) must satisfy the following conditions

$$RC \gg T_c \quad \text{and} \quad RC \ll T_m$$

$$\Rightarrow RC \gg \frac{2\pi}{\omega_c} \quad \text{--- (33)} \quad \Rightarrow RC \ll \frac{2\pi}{\omega_m} \quad \text{--- (34)}$$

$\because T = \frac{1}{f} = \frac{2\pi}{\omega}$
 $\omega = 2\pi f$

From (33) & (34),

$$\boxed{\frac{2\pi}{\omega_c} \ll RC \ll \frac{2\pi}{\omega_m}} \quad \text{--- (35)}$$

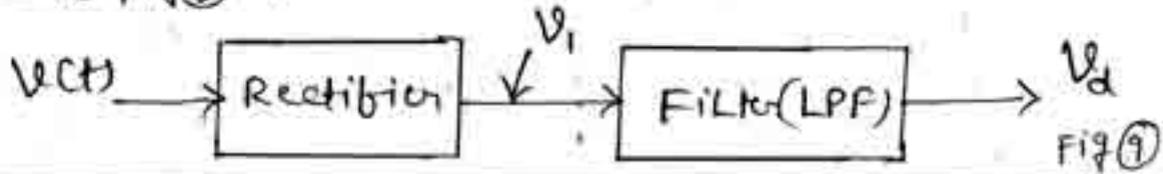
→ The detected envelope is,

$$V_d = V_c + M V_c \sin \omega_m t$$

Where, V_c → dc component can be easily removed by a simple RC LPF

$$\therefore \boxed{V_d = V_m \sin \omega_m t} \rightarrow \text{Message signal} \quad \left[\begin{array}{l} \because M = \frac{V_m}{V_c} \\ \Rightarrow M V_c = V_m \end{array} \right]$$

Note: (1) Fig (5) is equivalent to fig (9) shown below,



2) Frequency Modulation (FM):

The process of changing the frequency of carrier wave with respect to the instantaneous values of the message signal is called frequency modulation (FM)

→ Here the amplitude & phase of the carrier wave is kept constant.

→ When the message signal voltage is zero as at A, C, E, the carrier frequency is unchanged (same as frequency f_c)

→ When the message signal approaches its positive peak as at B, the carrier frequency is increased to maximum as shown by the closely spaced cycles. (High frequency)

→ When the message signal approaches its negative peak as at D, the carrier frequency is reduced to minimum as shown by the widely spaced cycles. (Low frequency)

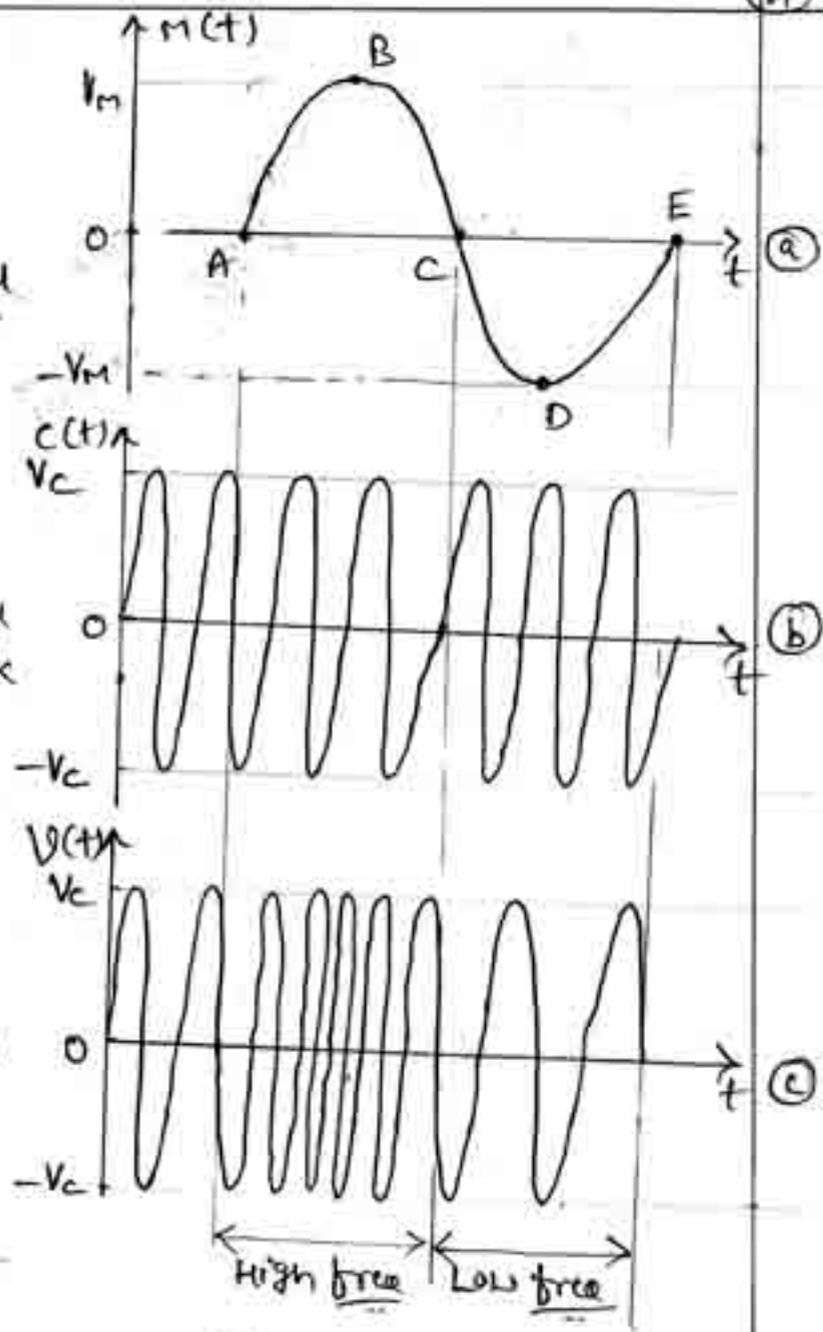


Fig 1

* FM Wave Equation @ Instantaneous Voltage of FM Wave:

Let the instantaneous frequency of the FM wave

$$f_i = f_c(1 + K V_m \cos \omega t) \quad \text{--- (1)}$$

Where, $f_c \rightarrow$ carrier frequency

$K \rightarrow$ Constant of Proportionality

Let the angular frequency of the FM wave is,

$$\omega_i = \frac{d\theta_i}{dt}$$

$$\Rightarrow \theta_i = \int_0^t \omega_i dt \quad \text{--- (4)}$$

$$\Rightarrow \theta_i = \int_0^t \omega_c (1 + K_v m \cos 2\pi f_m t) dt$$

$$\Rightarrow \theta_i = \int_0^t \omega_c dt + \int_0^t \omega_c K_v m \cos(2\pi f_m t) dt$$

$$\Rightarrow \theta_i = \omega_c t + \frac{\omega_c K_v m \sin(2\pi f_m t)}{2\pi f_m}$$

$$\Rightarrow \theta_i = \omega_c t + \frac{2\pi f_c K_v m \sin(2\pi f_m t)}{2\pi f_m}$$

$$\Rightarrow \theta_i = \omega_c t + \frac{K_f V_m \sin(2\pi f_m t)}{f_m}$$

$$\Rightarrow \theta_i = \omega_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t) \quad \text{--- (5)}$$

$$\Rightarrow \theta_i = \omega_c t + \beta \sin(2\pi f_m t) \quad \text{--- (6)}$$

The FM wave is,

$$V(t) = V_c \cos \theta_i \quad \text{--- (7)}$$

Using (6) in (7), we get

$$V(t) = V_c \cos [\omega_c t + \beta \sin(2\pi f_m t)] \quad \text{--- (8)}$$

$$V(t) = V_c \cos [2\pi f_c t + \beta \sin(2\pi f_m t)] \quad \text{--- (9)}$$

Let the message signal is,

$$m(t) = V_m \cos \omega_m t \quad \text{--- (2)}$$

Let the carrier signal is,

$$c(t) = V_c \cos \omega_c t \quad \text{--- (3)}$$

$$\omega_m = 2\pi f_m$$

$$\omega_c = 2\pi f_c$$

∴ From (1),

$$f_i = f_c (1 + K_v m \cos \omega_m t)$$

$$\Rightarrow 2\pi f_i = 2\pi f_c (1 + K_v m \cos \omega_m t)$$

$$\Rightarrow \omega_i = \omega_c (1 + K_v m \cos \omega_m t)$$

$$\text{Where } \omega_m = 2\pi f_m$$

Where $K_f = K_v f_c$
↓
Frequency sensitivity of FM (1)
Deviation constant

Where,
 $\Delta f = K_f V_m$
↓
Frequency deviation

$\beta = \frac{\Delta f}{f_m}$
↓
Modulation index

Modulation factor
(2) Phase deviation of FM wave

Eqn (8) & (9) are the expression for FM wave

Note: ① Modulation index for (FM): (β)

Modulation index is defined as the ratio of the frequency deviation to the modulating frequency.

ie $\beta = \frac{\Delta f}{f_m} \text{ or } \frac{\Delta \omega}{\omega_m} \text{ --- (10)}$

β is generally greater than 1 (measured in radians)

② Frequency spectrum:

FM consists of ① carrier (at f_c)

② Infinite number of sidebands

Upper side band frequencies at $f_c + f_s, f_c + 2f_s, \dots$
Lower side band frequencies at $f_c - f_s, f_c - 2f_s, \dots$

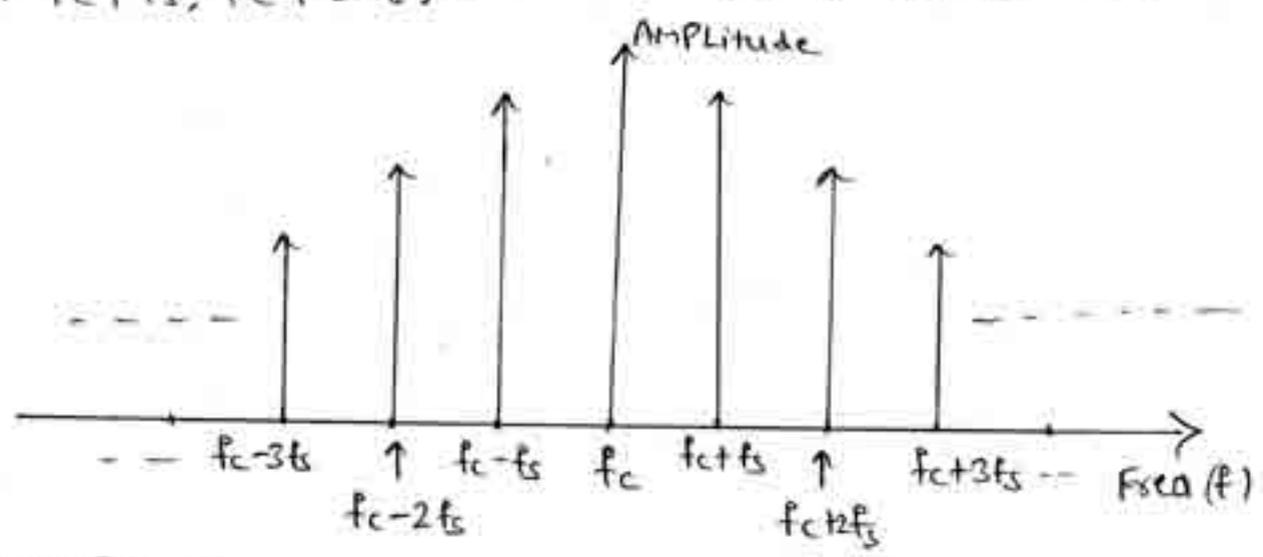


Fig ②: Frequency Spectrum of FM

③ Bandwidth:

From Carson's rule, the BW of FM is,

$BW = 2(\Delta f + f_m) \text{ or } 2\Delta f(1 + \frac{1}{\beta})$
 $BW = 2\beta f_m + 2f_m \text{ or } 2f_m(\beta + 1) \text{ --- (11)}$

- If $\Delta f > f_m$, it is Wideband FM
- If $\Delta f < f_m$, it is narrowband FM.

4) Total power in FM Wave (P_t)

$$P_t = \frac{V_c^2}{2} \sum_{n=-\infty}^{\infty} J_n^2(\beta) \quad \text{--- (12)}$$

Where,
 $J_n(\beta) \rightarrow$ Bessel function.

5) Frequency deviation

Consider eqn (1),

$$f_i = f_c (1 + K_f V_m \cos \omega_m t)$$

$$\Rightarrow f_i = f_c + K_f f_c V_m \cos \omega_m t$$

Maximum value of f_i occurs when $\cos \omega_m t = \pm 1$

$$\therefore f_i = f_c \pm K_f V_m \quad \text{--- (13)}$$

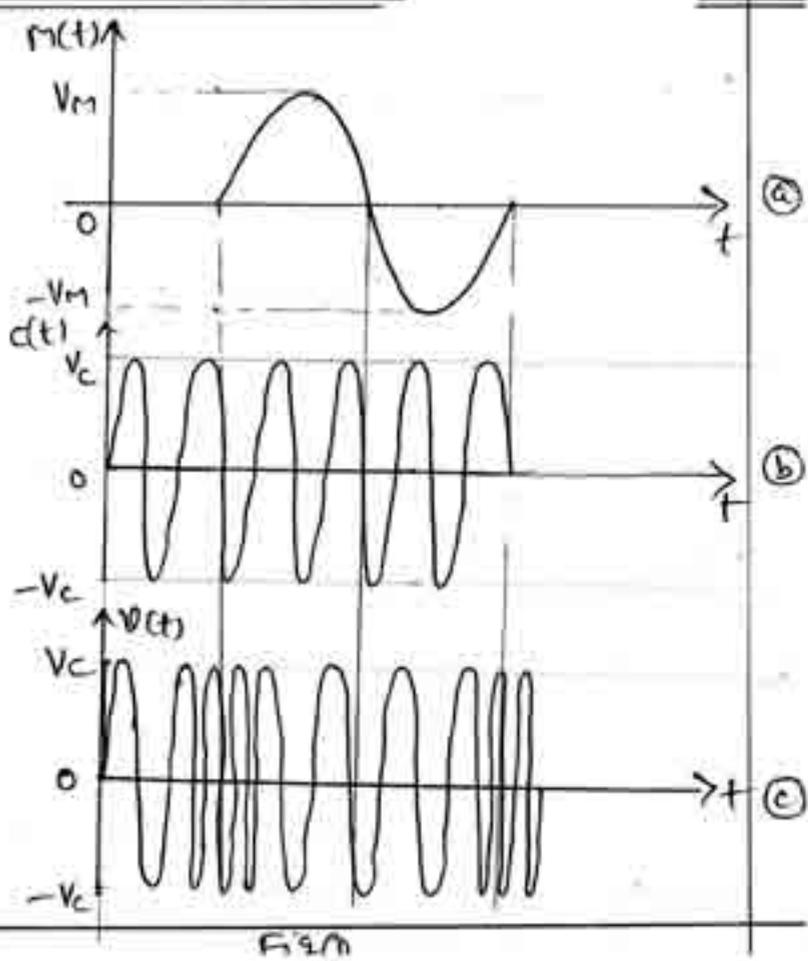
Comparing (13) with $f_i = f_c \pm \Delta f$ --- (14)

$\Delta f = K_f V_m$ \rightarrow Frequency deviation @
 Maximum frequency deviation.

3) Phase Modulation (PM):

The process of changing the phase of carrier wave with respect to the instantaneous values of the message signal is called Phase Modulation (PM)

\rightarrow Here the amplitude & frequency of the carrier wave is kept constant.



* PM Wave Equation @ Instantaneous Voltage of PM Wave

Let the message signal is,

$$m(t) = V_m \cos \omega_m t \quad \text{--- (1)}$$

[Angular freq of $m(t)$ is,
 $\omega_m = 2\pi f_m$]

Let the carrier signal is,

$$c(t) = V_c \cos \omega_c t \quad \text{--- (2)}$$

[Angular freq of $c(t)$ is,
 $\omega_c = 2\pi f_c$]

Let the instantaneous phase of the modulated carrier (PM wave) is,

$$\theta_i = \omega_c t + K_p m(t) \quad \text{--- (3)}$$

$$\Rightarrow \theta_i = \omega_c t + K_p V_m \cos \omega_m t$$

$$\Rightarrow \theta_i = \omega_c t + m_p \cos \omega_m t \quad \text{--- (3)}$$

[$K_p \rightarrow$ Frequency deviation (rad/V)
Phase Sensitivity of PM
Constant of Proportionality for PM]

The PM wave is,

$$v(t) = V_c \cos \theta_i \quad \text{--- (4)}$$

[Where, $m_p = K_p V_m$
 \downarrow
Modulation index]

Using (3) in (4), we get

$$v(t) = V_c \cos (\omega_c t + m_p \cos \omega_m t) \quad \text{--- (5)}$$

$$v(t) = V_c \cos (2\pi f_c t + m_p \cos 2\pi f_m t) \quad \text{--- (6)}$$

Eqn (5) & (6) are the expression for PM wave.

Note: @ $\omega_c t$ differentiation on (3) we get (ω_i at t)

$$\omega_i = \omega_c + K_p \frac{d}{dt} m(t)$$

$$\Rightarrow 2\pi f_i = 2\pi f_c + K_p \frac{d}{dt} m(t)$$

$$\Rightarrow f_i = f_c + \frac{K_p}{2\pi} \frac{d}{dt} m(t) \quad \text{--- (7)}$$

[$\therefore \frac{d\theta_i}{dt} = \omega_i$
 $\frac{d(t)}{dt} = 1$]

Eqn (7) is the instantaneous freq of PM wave

② Bandwidth

$BW = 2f_m (k_f V_m + 1) \text{ @ } 2f_m (m_f + 1)$

③ The average transmitted power of angle modulated wave (both FM & PM) is,

$P_{av} = \frac{1}{2} V_c^2$

$$\left[P_{av} = \frac{V_{cos}^2}{R} = \frac{\left(\frac{V_c}{\sqrt{2}}\right)^2}{R} = \frac{V_c^2}{2} \right]$$

$R = 1 \Omega$

* Advantages of AM (Merits):

- ① Bandwidth required is less (compared to FM & PM)
- ② Demodulation of AM is much simpler (compared to FM)
- ③ AM receivers are cheap. ⑤ AM is a linear process
- ④ wider coverage area (compared to FM)

* Disadvantages of AM @ Drawbacks @ Demerits:

- ① Transmission efficiency is poor ($\eta_{max} \leq 33.33\%$)
- ② Poor audio quality
- ③ Affected by noise
- ④ Limited operating range
- ⑤ Affected by electrical storms & other radio frequency interference

* Advantages of FM

- ① Better audio quality (compared to AM)
- ② Efficiency is high ($\eta \leq 100\%$)
- ③ More immune to noise (compared to AM)
- ④ operating range is quite large

- ⑤ No adjacent channel interference
- ⑥ Transmitted Power is constant & independent of modulation depth
- ⑦ Signal to noise ratio can be improved

* Disadvantages of FM

- ① Bandwidth required is much larger (compared to AM)
- ② Cannot be transmitted over long distances. (Area of reception is small)
- ③ Complex & expensive transmitting & receiving equipments.
- ④ Jamming of FM signal is easier than for AM.
- ⑤ Presence of tall buildings & land masses may limit the coverage & quality of FM.

* Comparison between AM & FM

AM	FM
① Amplitude of the carrier changes (Frequency & phase remains constant) (EM wave)	① Frequency of the carrier changes (Amplitude & phase remains constant) (EM wave)
② Modulation index varies from 0 to 1	② Modulation index is much greater than 1
③ More susceptible to noise	③ Less susceptible to noise
④ It has two side bands	④ It has infinite number of side bands
⑤ Transmitter & receiver are simple (IF for AM RX = 455 kHz)	⑤ Transmitter & receiver are more complex (IF for FM RX = 10.7 MHz)
⑥ Bandwidth = $2f_m$ (BW is less) (≈ 10 kHz)	⑥ BW = $2(\Delta f + f_m)$ (≈ 200 kHz) (BW is more compared to AM)
⑦ All transmitted power is not useful	⑦ All power is useful.
⑧ Poor audio quality	⑧ Better audio quality

- ⑨ More Prone to interference
- ⑩ Transmission efficiency is less ($\leq 33.33\%$)
- ⑪ Frequency ranges from 535 KHz to 1705 KHz
- ⑫ cover long distance
- ⑬ origin - 1870s (mid)
- ⑭ Propagation is by ground & sky waves
- ⑮ cannot handle weaker signals
- ⑯ AM wave is $V(t) = V_c [1 + m \sin(2\pi f_m t)] \sin(2\pi f_c t)$
- ⑰ Zero crossing is equidistant

- ⑨ Less Prone to interference
- ⑩ Better efficiency ($\leq 100\%$)
- ⑪ Frequency ranges from 88 MHz to 108 MHz.
- ⑫ cover small distances
- ⑬ origin - 1930s
- ⑭ Propagation is by space waves
- ⑮ can handle weaker signals
- ⑯ FM wave is $V(t) = V_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$
- ⑰ Zero crossing is not equidistant.

* Features of PM:

- ① Zero crossing of PM doesnot occur at regular intervals of time (For FM also)
- ② The envelope of PM wave is constant compared to AM (For FM also)
- ③ Instantaneous value of PM (& also FM) is a non-linear function of the information signal.
- ④ Phase of the carrier changes (frequency & amplitude remain constant)
- ⑤ PM wave is, $V(t) = V_c \cos(2\pi f_c t + M_p \cos(2\pi f_m t))$
- ⑥ PM is indirect method of producing FM
- ⑦ Modulation index, $M_p = K_p V_m$
- ⑧ BW = $2f_m(M_p + 1)$ (BW is more)
- ⑨ Average transmitted power, $P_{av} = \frac{1}{2} V_c^2$

- ⑩ PM is more immune to noise
- ⑪ Less prone to interference
- ⑫ Transmitter & Receiver are more complex
- ⑬ Visualization difficulty of message waveform

Problems

- ① A modulating signal consists of a symmetrical triangular wave, which has zero dc component & peak-to-peak voltage 11V. It is used to amplitude modulate a carrier of peak voltage 10V. Calculate the modulation index?

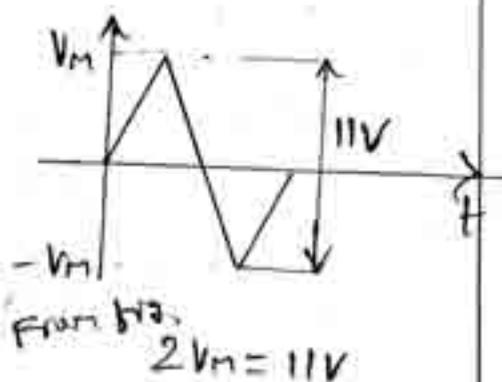
Sol: Peak Amplitude of modulating signal,

$$V_m = \frac{11}{2} = 5.5V$$

Modulation index, $m = \frac{V_m}{V_c}$

$$= \frac{5.5}{10}$$

$$\boxed{m = 0.55} //$$



Given,

$$V_c = 10V$$

- ② A carrier wave of frequency 10MHz & peak value 10V is amplitude modulated by a 5kHz sine wave of amplitude 6V. Determine the modulation index & amplitude of the side frequencies.

Sol: Given $f_c = 10 \times 10^6 \text{ Hz}$, $V_c = 10V$,
 $f_m = 5 \times 10^3 \text{ Hz}$, $V_m = 6V$.

Modulation Index

$$m = \frac{V_m}{V_c}$$

$$m = \frac{6}{10}$$

$$\boxed{m = 0.6} //$$

Amplitude of side frequencies

$$V_{SB} = \frac{mV_c}{2}$$

$$= \frac{0.6 \times 10}{2}$$

$$\boxed{V_{SB} = 3V} //$$

3) A broadcast radio transmitter radiates 10kW, when the modulation percentage is 60. How much of this is carrier power.

Sol: Given $P_t = 10 \times 10^3 \text{ W}$, $M = 60\% = 0.6$, $P_{ca} = ?$

$$\text{Lkt } P_t = P_{ca} \left(1 + \frac{M^2}{2} \right)$$

$$\Rightarrow P_{ca} = \frac{P_t}{1 + \frac{M^2}{2}} \\ = \frac{10 \times 10^3}{1 + \frac{0.6^2}{2}}$$

$$P_c = 8.47 \text{ kW} //$$

4) A radio transmitter radiates 10kW as carrier power or 8.5kW. Calculate modulation index.

Sol: Given $P_t = 10 \text{ kW}$, $P_{ca} = 8.5 \text{ kW}$ $M = ?$

$$\text{Lkt } P_t = P_{ca} \left(1 + \frac{M^2}{2} \right)$$

$$\Rightarrow M = \sqrt{2 \left(\frac{P_t}{P_{ca}} - 1 \right)} \\ = \sqrt{2 \left(\frac{10 \times 10^3}{8.5 \times 10^3} - 1 \right)}$$

$$M = 0.59 //$$

5) A 400W carrier is modulated to a depth of 7.5%. Calculate total power in the modulated wave.

Sol: Given $P_{ca} = 400 \text{ W}$, $M = 7.5\% = 0.075$, $P_t = ?$

$$\text{Lkt } P_t = P_{ca} \left(1 + \frac{M^2}{2} \right) = 400 \left(1 + \frac{0.075^2}{2} \right) = 401.12 \text{ W}$$

⑥ The antenna current of an AM transmitter is 8amps. When only the carrier is sent, but it increases to 8.93A, when the carrier is modulated by a single sine wave. Find Percentage modulation. Determine the antenna current when the Percent-Modulation changes to 0.8

Sol: Given $I_{car} = 8A$, $I_t = 8.93A$.

$$m = ?$$

Let

$$I_t = I_{car} \sqrt{1 + \frac{m^2}{2}}$$

$$\Rightarrow m = \sqrt{2 \left[\left(\frac{I_t}{I_{car}} \right)^2 - 1 \right]}$$

$$m = \sqrt{2 \left[\left(\frac{8.93}{8} \right)^2 - 1 \right]}$$

$$m = 0.7014 \text{ @ } 70.14\%$$

$$m = 0.8, I_t = ?$$

Let

$$I_t = I_{car} \sqrt{1 + \frac{m^2}{2}}$$

$$= 8 \sqrt{1 + \frac{0.8^2}{2}}$$

$$I_t = 9.191A //$$

⑦ Calculate the modulation index & Percentage modulation if instantaneous voltages of modulating signal & carrier are $40 \sin 4\pi t$ & $60 \sin 4\pi t$ respectively.

Sol: Given $m(t) = 40 \sin 4\pi t$ $v(t) = 60 \sin 4\pi t$

$$\Rightarrow V_m = 40$$

$$\Rightarrow V_c = 60$$

$$\therefore \text{modulation index, } m = \frac{V_m}{V_c} = \frac{40}{60} = 0.666$$

$$\% \text{ Modulation, } m\% = 66.66\%$$

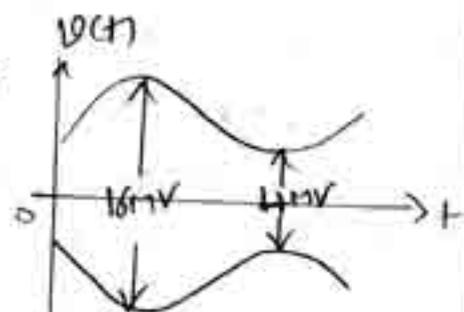
⑧ The maximum peak-to-peak voltage of an AM wave is 16mV & the minimum peak-to-peak voltage is 4mV.

Calculate the modulation factor.

Sol:

Given $2V_{max} = 16\text{mV}$, $2V_{min} = 4\text{mV}$

$\Rightarrow V_{max} = 8\text{mV}$, $V_{min} = 2\text{mV}$



∴ Modulation factor,

$$m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

$$= \frac{8 \times 10^{-3} - 2 \times 10^{-3}}{8 \times 10^{-3} + 2 \times 10^{-3}}$$

$$m = 0.6 \text{ @ } 60\%$$

⑨ A carrier of 100V & 1200kHz is modulated by a 50V , 1000Hz sine wave signal. Find the modulation factor.

Sol: Given, $V_c = 100\text{V}$, $V_m = 50\text{V}$

Modulation factor, $m = \frac{V_m}{V_c} = \frac{50}{100}$

$$m = 0.5 \text{ @ } 50\%$$

⑩ A 2500kHz carrier is modulated by audio signal with frequency span of $50 - 15000\text{Hz}$. What are the frequencies of lower & upper sidebands? What bandwidth of RF amplifier is required to handle the output?

Sol: Given $f_c = 2500 \times 10^3\text{Hz}$,

$f_{m1} = 50\text{Hz}$

$f_{m2} = 15000\text{Hz}$

$LSB_1 = f_c - f_{m1} = 2499.95\text{kHz}$

$LSB_2 = f_c - f_{m2} = 2485\text{kHz}$

$USB_1 = f_c + f_{m1} = 2500.05\text{kHz}$

$USB_2 = f_c + f_{m2} = 2515\text{kHz}$

$BW_1 = USB_1 - LSB_1$

$BW_2 = USB_2 - LSB_2$

$$BW_1 = 100\text{KHz} \quad (2f_m) \quad | \quad BW_2 = 30\text{KHz} \quad (2f_{m2})$$

Upper side band ranges from 2500.05 KHz - 2515 KHz

Lower side band ranges from 2499.95 KHz - 2485 KHz

Bandwidth ranges from 100KHz - 30KHz

10 An AM wave is given by.

$$V = 5(1 + 0.6 \cos 6280t) \sin 211 \times 10^4 t \text{ Volts}$$

(i) What are the minimum & max amplitudes of the AM wave? (ii) What frequency components are contained in the modulated wave & what is the amplitude of each component?

Sol: Given $V = 5(1 + 0.6 \cos 6280t) \sin 211 \times 10^4 t$ — (1)

Comparing it with standard AM wave.

$$V(t) = V_c(1 + m \cos \omega_m t) \sin \omega_c t \text{ — (2)}$$

From (1) & (2), we get

$V_c = 5V$ $M = 0.6$ $\Rightarrow \frac{V_m}{V_c} = 0.6$ $\Rightarrow V_m = 0.6 \times 5$ $V_m = \underline{3V}$	$\omega_m = 6280$ $2\pi f_m = 6280$ $\Rightarrow f_m = \frac{6280}{2\pi}$ $f_m \approx 1\text{KHz}$	$\omega_c = 211 \times 10^4$ $2\pi f_c = 211 \times 10^4$ $\Rightarrow f_c = \frac{211 \times 10^4}{2\pi}$ $f_c = 335.816\text{KHz}$
--	--	---

(i) Minimum & Max amplitudes of AM wave

$$V_{min} = V_c - V_m = 5 - 3 = \underline{2V}$$

$$V_{max} = V_c + V_m = 5 + 3 = \underline{8V}$$

(ii) Free components & their amplitudes of each component

AM wave will contain three frequency components

- ① carrier: $f_c = 335.816 \text{ KHz}$
- ② LSB: $f_{LSB} = f_c - f_m = 335.816 \text{ KHz} - 1 \text{ KHz} = 334.816 \text{ KHz}$
- ③ USB: $f_{USB} = f_c + f_m = 335.816 \times 10^3 + 1 \times 10^3 = 336.816 \text{ KHz}$

Amplitude of three components

- ① carrier: $V_c = 5 \text{ V}$
- ② LSB: $V_{LSB} = \frac{mV_c}{2} = \frac{3}{2} = 1.5 \text{ V}$
- ③ USB: $V_{USB} = \frac{mV_c}{2} = \frac{3}{2} = 1.5 \text{ V}$

12) A FM voltage wave is given by,

$$e = 12 \cos(6 \times 10^8 t + 5 \sin 1250 t)$$

- Find (i) carrier frequency (ii) signal frequency
- (iii) modulation index (iv) maximum frequency deviation
- (v) power dissipated by the FM wave in load resistor
- with BW.

Sol:
 Given $e = 12 \cos(6 \times 10^8 t + 5 \sin 1250 t)$ — (1)
 Comparing with $v(t) = V_c \cos(\omega_c t + \beta \sin \omega_m t)$ — (2)
 From ① & ②, $V_c = 12 \text{ V}$, $\omega_c = 6 \times 10^8$, $\beta = 5$, $\omega_m = 1250$

(i) f_c $f_c = \frac{\omega_c}{2\pi} = \frac{6 \times 10^8}{2\pi} = 95.5 \text{ MHz}$

(ii) f_m $f_m = \frac{\omega_m}{2\pi} = \frac{1250}{2\pi} = 199 \text{ Hz}$

(iii) β $\beta = 5\%$

(iv) Δf $\Delta f = \beta f_m = 5 \times 199 = 995 \text{ Hz}$

(v) P

$$P = \frac{V_{rms}^2}{R}$$

$$= \frac{(V_c/\sqrt{2})^2}{10}$$

$$= 7.2 \text{ W}$$

(vii) $BW = 2(\Delta f + f_m) = 2(995 + 199) = 2.388 \text{ KHz}$

(13) In FM broadcasting $\delta_{\text{max}} = 75 \text{ KHz}$, & modulation frequency 15 KHz , what is deviation ratio

Sol. Given $\delta_{\text{max}} = \Delta f = 75 \text{ KHz}$, $f_m = 15 \text{ KHz}$

B $\beta = \frac{\Delta f}{f_m} = \frac{75 \text{ K}}{15 \text{ K}} = 5\%$ (Deviation ratio)

(14) For an AM wave, the modulated & minimum amplitudes are 500 mV & 300 mV respectively, find Modulation index & Percentage of Modulation

Sol. Given $V_{\text{max}} = 500 \text{ mV}$, $V_{\text{min}} = 300 \text{ mV}$.

We have, $M = \frac{V_{\text{max}} - V_{\text{min}}}{V_{\text{max}} + V_{\text{min}}} = \frac{500 \times 10^{-3} - 300 \times 10^{-3}}{500 \times 10^{-3} + 300 \times 10^{-3}}$

Modulation index $\rightarrow \boxed{M = 0.25}$ // Percentage of Modulation = 25% //

(15) In a FM str, audio frequency is 500 Hz , AF Voltage is 2 V & the ~~the~~ frequency deviation is 4.8 KHz .

Find the modulation index.

Sol. Given $f_m = 500 \text{ Hz}$, $V_m = 2 \text{ V}$, $\Delta f = 4.8 \times 10^3 \text{ Hz}$, $\beta = ?$

We have, $\beta = \frac{\Delta f}{f_m} = \frac{4.8 \times 10^3}{500} = \underline{\underline{9.6}}$

(16) What is the antenna height required for AM with carrier frequency $f = 4 \text{ MHz}$? [$\because \lambda = c/f$]

Sol. Given $f(f_c) = 4 \text{ MHz}$

Height (size) of antenna; $h = \frac{\lambda}{4} = \frac{c}{f \times 4} = \frac{3 \times 10^8}{4 \times 10^6 \times 4} = \underline{\underline{18.75 \text{ m}}}$

17) Find the maximum power efficiency of an AM Modulator.

Sol: L.k.t carrier power $P_{con} = \frac{V_{con}^2}{R} = \frac{(V_c/\sqrt{2})^2}{R} = \frac{V_c^2}{2R}$ — (1)
 (Does not contain information)

Power contained in sidebands $P_{SB} = P_{LSB} + P_{USB}$
 (contains information)

$$= \frac{V_{LSB}^2}{R} + \frac{V_{USB}^2}{R}$$

$$= \frac{(\frac{mV_c}{2}/\sqrt{2})^2}{R} + \frac{(\frac{mV_c}{2}/\sqrt{2})^2}{R}$$

$$= \frac{m^2 V_c^2}{4R} \quad \text{--- (2)}$$

Now,

Power efficiency $\eta_p = \frac{P_{SB}}{P_{con}} = \frac{\frac{m^2 V_c^2}{4R}}{\frac{V_c^2}{2R}} = \frac{m^2}{2}$

maximum value of $m = 1$

∴ maximum power efficiency $(\eta_p)_{max} = \frac{1}{2} = 0.5 @ 50\%$

18) The total power content of an AM signal is 1.5 kW. Determine the power being transmitted at the carrier freq & at each of the sidebands when the modulation is 50%.

Sol: Given $P_t = 1.5 \times 10^3 W$, $m = 50\% = 0.5$, $P_{con} = ?$,
 $P_{USB} = ?$, $P_{LSB} = ?$

L.k.t $P_t = P_{con} + P_{USB} + P_{LSB}$ — (1) $P_{con} \left(1 + \frac{m^2}{2}\right)$ — (2)

$$\Rightarrow P_{con} = \frac{P_t}{1 + \frac{m^2}{2}} = \frac{1.5 \times 10^3}{1 + \frac{0.5^2}{2}} = \underline{\underline{1.333 \text{ kW}}}$$

NOW FROM ①.

$$\begin{aligned} P_{USB} + P_{LSB} &= P_t - P_{con} \\ &= 1.5 \times 10^3 - 1.333 \times 10^3 \\ &= 167 \text{ W} \end{aligned}$$

$$\Rightarrow P_{USB} = P_{LSB} = \frac{167}{2} = \underline{\underline{83.5 \text{ W}}} \quad [\because P_{USB} = P_{LSB}]$$

19) An AM wave has a power content of 1000W at its carrier frequency. Determine the power of each sideband for 70% modulation.

Sol: Given $P_{con} = 1000 \text{ W}$, $m = 70\% = 0.7$, $P_{LSB} = P_{USB} = ?$

$$\text{Wkt } P_{LSB} = P_{USB} = P_{con} \frac{m^2}{4} = \frac{1000 \times 0.7^2}{4} = \underline{\underline{122.5 \text{ W}}}$$

20) Determine modulation factor of signal if each of the sidebands contains 100W. Given total power of an AM wave = 800W.

Sol: Given $m = ?$, $P_{LSB} = P_{USB} = 100 \text{ W}$, $P_t = 800 \text{ W}$

$$\text{Wkt } P_t = P_{con} + P_{USB} + P_{LSB}$$

$$\Rightarrow P_{con} = P_t - P_{USB} - P_{LSB} = 800 - 100 - 100$$

$$\boxed{P_{con} = 600 \text{ W}}$$

Also we have $P_t = P_{con} \left(1 + \frac{m^2}{2}\right)$

$$\Rightarrow m = \sqrt{2 \left(\frac{P_t}{P_{con}} - 1\right)} = \sqrt{2 \left(\frac{800}{600} - 1\right)} = 0.816 \text{ @ } 81.6\%$$

1) A sinusoidal carrier voltage of amplitude 150V is amplitude modulated by a signal of freq 1KHz resulting in maximum modulated carrier of 220V. Calculate modulation factor

Sol: Given $V_c = 150V$, $f_m = 1 \times 10^3 Hz$, $V_{max} = 220V$

$m = ?$

Let here,

$$m = \frac{V_{max} - V_{min}}{V_{max} + V_{min}}$$

$$= \frac{220 - 80}{220 + 80} \left[\begin{array}{l} \because V_{min} = \\ 2V_c - V_{max} \\ V_{min} = 80V \end{array} \right]$$

$m = 0.466 @ 46.6\%$

or We have

$$m = \frac{V_m}{V_c} \quad \left[\begin{array}{l} V_m = V_{max} - V_c \\ V_m = 220 - 150 \\ V_m = 70 \end{array} \right]$$

$$= \frac{70}{150}$$

$m = 0.466 @ 46.6\%$

2) An AM transmitter radiates 10kW with the carrier unmodulated & 11kW when the carrier is sinusoidally modulated. Calculate the modulation index. If another sine wave with modulation index 50% is transmitted simultaneously, determine total power.

Sol: Given $P_{car} = 10kW$, $P_t = 11kW$, $m_1 = ?$

$m_2 = 50\% @ 0.5$, $P_{total} = ?$

Let $P_t = P_{car} \left(1 + \frac{m_1^2}{2}\right)$

$$\Rightarrow m_1 = \sqrt{2 \left(\frac{P_t}{P_{car}} - 1 \right)} = \sqrt{2 \left(\frac{11 \times 10^3}{10 \times 10^3} - 1 \right)} = \underline{0.44}$$

Let $m = \sqrt{m_1^2 + m_2^2} = \sqrt{0.44^2 + 0.5^2} = 0.66$ (Total Modulation Index)

Total Power, $P_{total} = P_{car} \left(1 + \frac{m^2}{2}\right) = 10 \times 10^3 \left(1 + \frac{0.66^2}{2}\right) = \underline{12.178 kW}$

23) The antenna current of an AM transmitter, modulated to 50% by a sine wave is 10A. It increases to 11A as a result of simultaneous modulation by another sine wave. What is the modulation index of second wave?

Sol: Given $m_1 = 50\% = 0.5$, $I_{t1} = 10A$, $I_t = 11A$, $m_2 = ?$

Let $I_{car} = \frac{I_{t1}}{\sqrt{1 + \frac{m_1^2}{2}}} = \frac{10}{\sqrt{1 + \frac{0.5^2}{2}}} = \underline{\underline{9.428A}}$

Now $I_t = I_{car} \sqrt{1 + \frac{m^2}{2}}$
(Total)

$\Rightarrow m = \sqrt{2 \left[\left(\frac{I_t}{I_{car}} \right)^2 - 1 \right]} = \sqrt{2 \left[\left(\frac{11}{9.428} \right)^2 - 1 \right]} = 0.85$

Let $m^2 = \sqrt{m_1^2 + m_2^2}$

$\Rightarrow m_2^2 = \sqrt{m^2 - m_1^2} = \sqrt{0.85^2 - 0.5^2} = \underline{\underline{0.68}}$

4) An audio signal is $15 \sin(2\pi 2000t)$ Amplitude modulates a carrier signal $60 \sin(2\pi 100,000t)$. Determine

- (i) Modulation factor
- (ii) Frequency of signal & carrier &
- (iii) Frequency components of modulated wave
- (iv) Plot the spectrum

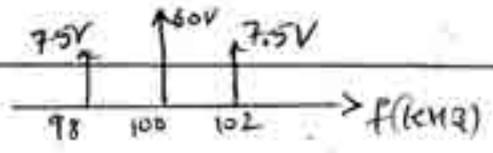
Sol: Given $m(t) = 15 \sin(2\pi 2000t)$, $c(t) = 60 \sin(2\pi 100,000t)$

comparing with $m(t) = V_m \sin(\omega_m t)$, comparing with $c(t) = V_c \sin(\omega_c t)$

$\Rightarrow \boxed{V_m = 15}$, $\omega_m = 2\pi f_m = 2\pi 2000$, $\boxed{V_c = 60}$, $\boxed{f_c = 100,000 \text{ Hz}}$
 $\Rightarrow \boxed{f_m = 2000 \text{ Hz}}$

(i) $M = \frac{V_m}{V_c} = \frac{15}{60} = 0.25$ (ii) $f_m = 2 \text{ kHz}$
 $f_c = 100 \text{ kHz}$

(iii) Carrier $f_c = 100\text{kHz}$



USB $f_{USB} = f_c + f_m = 100\text{kHz} + 2\text{kHz} = \underline{102\text{kHz}}$

LSB $f_{LSB} = f_c - f_m = 100\text{kHz} - 2\text{kHz} = \underline{98\text{kHz}}$

25) A Bandwidth of 20MHz is available for AM transmitters. If maximum audio freq to be used is 5kHz , how many stations can broadcast?

Sol: Given $f_m = 5\text{kHz}$

$$\Rightarrow BW = 2f_m$$

$$= 2 \times 5\text{K}$$

$$BW = 10\text{kHz}$$

Bandwidth required by each station

\therefore NO of stations

$$N = \frac{20\text{MHz}}{10\text{kHz}}$$

$$N = 2000 \text{ Channels @ Stations}$$

26) An FM signal has a resting freq of 105MHz & highest freq of 105.03MHz when modulated by a signal of freq 5kHz . Determine

- (i) freq deviation (ii) carrier swing (iii) Modulation index (iv) Lowest freq of FM wave (v) Bandwidth of signal.

Sol: Given $f_c = 105\text{MHz}$, $f_h = 105.03\text{MHz}$, $f_m = 5\text{kHz}$

① Δf $\Delta f = f_h - f_c = 105.03 \times 10^6 - 105 \times 10^6 = \underline{30\text{kHz}}$

② carrier swing carrier swing = $2\Delta f = 60\text{kHz} //$

③ Modulation index $\beta = \frac{\Delta f}{f_m} = \frac{30 \times 10^3}{5 \times 10^3} = \underline{6}$

④ Lowest freq of FM wave $f_L = f_c - \Delta f = 105 \times 10^6 - 30 \times 10^3$

$f_c = 104.97 \text{ MHz}$

① BL $BL = 2(\Delta f + f_m) = 2(30 \times 10^3 + 5 \times 10^3) = 70 \text{ kHz}$

② When the modulating frequency in FM is 600 Hz & modulating voltage is 3V, the modulation index is 10. Calculate the maximum deviation. What is the modulation index when the modulating frequency is reduced to 400 Hz & the modulating voltage is changed to 5V.

Sol.

① Given $f_m = 600 \text{ Hz}$, $V_m = 3 \text{ V}$, $\beta = 10$

W.K.T Maximum deviation,

$\Delta f = \beta f_m = 10 \times 600 = 6 \text{ kHz}$ ($\because \beta = \frac{\Delta f}{f_m}$)

Also $K_f = \frac{\Delta f}{V_m} = \frac{6 \times 10^3}{3} = 2 \text{ kHz/V}$ ($\because \Delta f = K_f V_m$)

② $f_m = 400 \text{ Hz}$, $V_m = 5 \text{ V}$, $\beta = ?$

We have, $\Delta f = K_f V_m = 2 \times 10^3 \times 5 = 10 \text{ kHz}$

Now $\beta = \frac{\Delta f}{f_m} = \frac{10 \times 10^3}{400} = 25$

③ A carrier wave of amplitude 3V & frequency 10 MHz is frequency modulated by a sinusoidal signal of 6V & freq 5 kHz. Find the frequency deviation & BW. Given constant = 1 kHz/Volt.

Sol.

Given $V_c = 3 \text{ V}$, $f_c = 10 \times 10^6 \text{ Hz}$, $V_m = 6 \text{ V}$, $f_m = 5 \text{ kHz}$.

$\Delta f = ?$, $BL = ?$, $K_f = 1 \text{ kHz/V}$

$\Delta f = K_f V_m = 1 \times 10^3 \times 6 = 6 \text{ kHz}$ | $BL = 2(\Delta f + f_m) = 11 \text{ kHz}$

29) An AM broadcasting station broadcasts with an average transmitted power of 200W at a modulation index of 60%. Find the transmission power efficiency & the average power in carrier.

Sol: $P_t = 200W, m = 60\% = 0.6$

Carrier Power $P_c = \frac{P_t}{1 + \frac{m^2}{2}} = \frac{200}{1 + \frac{0.6^2}{2}} = 169.49W$

Transmission power efficiency

$\eta = \frac{m^2}{2 + m^2} = 0.1525 @ \underline{\underline{15.25\%}}$

30) The carrier frequency in an FM modulator is 10MHz & modulating freq is 20KHz. What are the first three upper sideband & lower sideband freq?

Sol Given $f_m = 20KHz, f_c = 10MHz$

Upper sideband frequencies

① $f_{us01} = f_c + f_s = 10 \times 10^6 + 2 \times 10^3 = 10.002MHz = f_{us01}$

② $f_{us02} = f_c + 2f_s = 10 \times 10^6 + 2 \times 2 \times 10^3 = 10.004MHz = f_{us02}$

③ $f_{us03} = f_c + 3f_s = 10 \times 10^6 + 3 \times 2 \times 10^3 = 10.006MHz = f_{us03}$

Lower sideband frequencies

④ $f_{ls01} = f_c - f_s = 10 \times 10^6 - 2 \times 10^3 = 9.998MHz //$

⑤ $f_{ls02} = f_c - 2f_s = 10 \times 10^6 - 2 \times 2 \times 10^3 = 9.996MHz //$

⑥ $f_{ls03} = f_c - 3f_s = 10 \times 10^6 - 3 \times 2 \times 10^3 = 9.994MHz //$

31) In an FM SLM, when the audio freq is 500Hz & the AF voltage is 2.4V, the frequency deviation is

4.8 KHz. If the AF Voltage is now increased to 7.2V, what is the new frequency deviation? If the AF voltage is raised to 10V while the AF is dropped to 200Hz, what is the deviation? Find the modulation index in each case.

Sol: Given (a) $f_m = 500 \text{ Hz}$, $V_m = 2.4 \text{ V}$, $\Delta f_1 = 4.8 \text{ KHz}$, $\beta_1 = ?$

(b) $V_m = 7.2 \text{ V}$, $\Delta f_2 = ?$, $\beta_2 = ?$

(c) $V_m = 10 \text{ V}$, $\Delta f_3 = ?$, $f_m = 200 \text{ Hz}$, $\beta_3 = ?$

(a)
$$\beta_1 = \frac{\Delta f_1}{f_m} = \frac{4.8 \times 10^3}{500} = \underline{\underline{9.6}}$$

(b)
$$\Delta f_2 = K_f V_m = 2 \times 10^3 \times 7.2 = \underline{\underline{14.4 \text{ KHz}}}$$

$$\beta_2 = \frac{\Delta f_2}{f_m} = \frac{14.4 \times 10^3}{500} = \underline{\underline{28.8}}$$

$$K_f = \frac{\Delta f}{V_m} = \frac{4.8 \times 10^3}{2.4} = 2 \text{ KHz/V}$$

(c)
$$\Delta f_3 = K_f V_m = 2 \times 10^3 \times 10 = \underline{\underline{20 \text{ KHz}}}$$

$$\beta_3 = \frac{\Delta f_3}{f_m} = \frac{20 \times 10^3}{200} = \underline{\underline{100\%}}$$

2) A carrier wave of 10V & 10MHz is amplitude modulated by AF signal of 4KHz. Write the eqn of AM wave with modulation of 50%.

Sol: Given $V_c = 10 \text{ V}$, $f_c = 10 \text{ MHz}$, $f_m = 4 \text{ KHz}$, $m = 50\% = 0.5$

AM wave is,

$$v(t) = (V_c + V_m \sin \omega_m t) \sin \omega_c t$$

$$V(t) = [10 + 5 \sin(2\pi \times 4 \times 10^3 t)] \sin(2\pi \times 10 \times 10^6 t)$$

(27)

$$V(t) = V_c (1 + m \sin \omega_m t) \sin \omega_c t$$

$$V(t) = 10 [1 + 0.5 \sin(2\pi \times 4 \times 10^3 t)] \sin(2\pi \times 10 \times 10^6 t)$$

(28)

$$V(t) = V_c \sin \omega_c t + \frac{m V_c}{2} \cos(\omega_c - \omega_m) t - \frac{m V_c}{2} \cos(\omega_c + \omega_m) t$$

$$V(t) = 10 \sin(2\pi \times 10 \times 10^6 t) + \frac{0.5 \times 10}{2} \cos(2\pi \times 10 \times 10^6 - 2\pi \times 4 \times 10^3) t - \frac{0.5 \times 10}{2} \cos(2\pi \times 10 \times 10^6 + 2\pi \times 4 \times 10^3) t$$

$$\Rightarrow V(t) = 10 \sin(2\pi \times 10^7 t) + 2.5 \cos[2\pi(10^7 - 4 \times 10^3) t] - 2.5 \cos[2\pi(10^7 + 4 \times 10^3) t]$$

$$\begin{aligned} m &= \frac{V_m}{V_c} \\ \Rightarrow V_m &= m V_c \\ &= 0.5 \times 10 \\ V_m &= 5V \\ \omega_m &= 2\pi f_m \\ \omega_c &= 2\pi f_c \end{aligned}$$

(33) A 20 MHz, 3V carrier is modulated by a 1 kHz signal. The maximum frequency deviation is 10 kHz & the same modulation index is used for both FM & PM. Write expressions for FM & PM

Sol: Given $f_c = 20 \times 10^6 \text{ Hz}$, $V_c = 3V$, $f_m = 1 \times 10^3 \text{ Hz}$, $\Delta f = 10 \times 10^3 \text{ Hz}$

$$\beta = m_p = \frac{\Delta f}{f_m} = \frac{10 \times 10^3}{1 \times 10^3} = 10$$

FM wave, $V(t) = V_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)]$

$$\Rightarrow V(t) = 3 \cos[2\pi \times 20 \times 10^6 t + 10 \sin(2\pi \times 10^3 t)]$$

PM wave, $V(t) = V_c \cos(\omega_c t + m_p \cos \omega_m t)$

$$\Rightarrow V(t) = 3 \cos[2\pi \times 20 \times 10^6 t + 10 \cos(2\pi \times 10^3 t)]$$

$$(\because \omega_m = 2\pi f_m \text{ \& } \omega_c = 2\pi f_c)$$